WORKSHOP ARITHMETIC FRANCE-JAPAN
日本 x フランス 数論幾何学 2025

Komaba Campus, The University of Tokyo, Japan
From April 14 to April 18, 2025

Organizers
Naoki IMAI, University of Tokyo
Francesco LEMMA, IMJ-PRG
Tadashi OCHIAI, Institute of Science Tokyo

MONDAY - APR. 14, 2025

9:00	**Welcome & Registration**
9:15	Opening words
9:30-10:15	S. MORRA :: Local models for Galois deformation rings (1)
10:45-11:30	S. MORRA :: Local models for Galois deformation rings (2)
	Lunch break
13:15-14:00	Y. KEZUKA:: On the Birch and Swinnerton-Dyer conjecture for elliptic curves of the form $x^3 + y^3 = N$ (1)
14:15-15:00	Y. KEZUKA:: On the Birch and Swinnerton-Dyer conjecture for elliptic curves of the form $x^3 + y^3 = N$ (2)
15:30-16:30	T. TAKAMATSU :: Arithmetic finiteness of certain Fano varieties

TUESDAY - APR. 15, 2025

9:30-10:15	Y. HOSHI :: Anabelian Geometry of Hyperbolic Curves, Progress on Section Conjecture (1)
10:30-11:15	Y. HOSHI :: Anabelian Geometry of Hyperbolic Curves, Progress on Section Conjecture (2)
11:45-12:30	A. MAKSOUD :: Iwasawa theory and the geometry of the eigencurve (1)
	Lunch break
14:00-14:45	A. MAKSOUD :: Iwasawa theory and the geometry of the eigencurve (2)
15:15-16:00	S. KELLY :: Motivic cohomologies for qcqs schemes (1)
16:15-17:00	S. KELLY :: Motivic cohomologies for qcqs schemes (2)

WEDNESDAY - APR. 16, 2025

9:30-10:15	Y. YATAGAWA :: Index formula of partially logarithmic characteristic cycles (1)
10:30-11:15	Y. YATAGAWA :: Index formula of partially logarithmic characteristic cycles (2)
11:45-12:30	J. RODRIGUES JACINTO :: Solid locally analytic representations (1)
	Lunch break
14:00-14:45	J. RODRIGUES JACINTO:: Solid locally analytic representations (2)
15:15-16:00	E. LEPAGE:: Anabelian reconstruction of Berkovich spaces through resolution of non-singularities and Grothendieck's p-adic anabelian conjecture (1)
16:15-17:00	E. LEPAGE :: Anabelian reconstruction of Berkovich spaces through resolution of non-singularities and Grothendieck's p-adic anabelian conjecture (2)
18:00-20:00	**Banquet**

Thursday - Apr. 17, 2025

9:30-10:15 Y. Mieda :: On Fargues-Scholze local Langlands correspondence for some supercuspidal representations of $Sp(6)$ (1)

10:30-11:15 Y. Mieda :: On Fargues-Scholze local Langlands correspondence for some supercuspidal representations of $Sp(6)$ (2)

11:45-12:30 E. Lecouturier :: Eisenstein cocycles for imaginary quadratic fields (1)

Lunch break

14:00-14:45 E. Lecouturier:: Eisenstein cocycles for imaginary quadratic fields (2)

15:15-16:00 V. Ertl :: Different approaches to Hyodo-Kato theory (1)

16:15-17:00 V. Ertl :: Different approaches to Hyodo-Kato theory (2)

Friday - Apr. 18, 2025

9:30-10:30 K. Sawada :: Families preserving isomorphisms via techniques in anabelian geometry

10:45-11:30 N. Abe :: Reducibility of p-adic Banach principal series representations (1)

11:45-12:30 N. Abe :: Reducibility of p-adic Banach principal series representations (2)

Lunch break

14:00-17:00 Free discussion

Venue. All talks take place in the "Lecture Hall" of the Graduate School of Mathematical Sciences Bldg, Komaba Campus, University of Tokyo (Address: 3-8-1 Komaba, Meguro-ku, Tokyo 153-8902).

For local-global maps and instructions, see `https://www.c.u-tokyo.ac.jp/eng_site/info/about/visitors/maps-directions/`

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Titles and Abstracts

Monday - April 14

9:30–11:30 • Local models for Galois deformation rings

Stefano Morra *:: Paris Saint-Denis University, France*

The theory of local models for Shimura variety is a vast subject, introduced by Deligne, Pappas, Rapoport and many others in the early 90's, pursuing the idea that the singularities of the mod p reduction of Shimura varieties can be controlled by group-theoretic objects closely related to Grassmannians.

On an apparently completely different topic, the theory of deformation of Galois representations over $\mathbb{Z}_p$-algebras turned out to be a fundamental tool in the proof of deep arithmetic statements, the Tanyiama-Shimura-Weil conjecture through the Taylor–Wiles method being one of the most celebrated example.

In the early 2000's, the theory developed by Breuil and Kisin showed that certain particularly relevant deformation rings – namely deformations of representations of $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ with constraints from p-adic Hodge theory – can be described in terms of local models. This had immense consequences as the proof of vast generalizations of the Tanyiama-Shimura-Weil conjecture (modularity statements), the Breuil–Mézard conjecture and the opening of the p-adic local Langlands program.

In the first of my talks I will give, following [Mor24] a more detailed account on the interplay between local models, Galois representations, and how these are relevant to approach problems in the mod p Langlands program, all this without entering the details but rather giving exemples and key results from their interactions.

In the second of my talks I will present in more detail, following [LHMM23], how the theory of local models gives a complete control on tamely potentially Barsotti–Tate deformations, these latter being the gateway to the more general theory of Galois deformations with p-adic Hodge theory conditions.

References

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13:15–15:00 • On the Birch and Swinnerton-Dyer conjecture for elliptic curves of the form $x^3 + y^3 = N$

Yukako Kezuka *:: Kanazawa University, Japan*

The Birch–Swinnerton-Dyer conjecture is one of the most celebrated open problems in number theory. In this talk, I will discuss some recent progress on the study of this conjecture for elliptic curves E of the form $x^3 + y^3 = N$, where N is a cube-free positive integer. These curves are cubic twists of the Fermat elliptic curve $x^3 + y^3 = 1$ and admit complex multiplication by the ring of integers of $\mathbb{Q}(\sqrt{-3})$.

In the first talk, I will introduce these curves and explain how divisibility properties of central L-values provide insights into their arithmetic. In particular, I will describe an interesting relation between the 3-part of their Tate–Shafarevich group and the number of prime divisors of N that remain inert in $\mathbb{Q}(\sqrt{-3})$.

In the second talk, I will present joint work with Yongxiong Li, where we study in more detail the cases when $N = 2p$ or $2p^2$ for an odd prime p congruent to 2 or 5 modulo 9. For these curves, we establish the 3-part of the Birch–Swinnerton-Dyer conjecture and explore a connection between the ideal class group of $\mathbb{Q}(\sqrt[3]{p})$ and the 2-Selmer group of E. This connection can be used to study non-triviality of the 2-part of their Tate–Shafarevich group.

References

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15:30–16:30 • Arithmetic finiteness of certain Fano varieties

. . . . *Teppei Takamatsu :: Kyoto University, Japan*

As an analogue of the problem of how many integer solutions a given equation has, the question of how many algebraic varieties exist over a ring of integers has been a subject of classical study. For example, the following theorem, proved by Faltings and Zarhin, concerns the finiteness of abelian varieties over a rings of integer and is well known as the Shafarevich conjecture (for abelian varieties):

> Isomorphism classes of abelian varieties of a fixed dimension over a fixed number field admitting good reduction away from a fixed finite set of finite places are finite.

See the book [FWG+] for this theorem and its application to the Mordell conjecture.

The Shafarevich conjecture is expected to hold not only for abelian varieties but also for a broader class of varieties. For example, the Shafarevich conjecture for del Pezzo surfaces (= Fano varieties of dimension 2) was proved by Scholl ([Sch85]).

In this talk, we consider analogous problems for certain Fano varieties of dimension greater than 2. This talk is based on joint work with Tetsushi Ito, Akihiro Kanemitsu, and Yuuji Tanaka (including [IKTT24]).

References

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[Sch85] Anthony J. Scholl, A finiteness theorem for del Pezzo surfaces over algebraic number fields, *J. London Math. Soc. (2)* **32** (1985), no. 1. 31–40.

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TUESDAY - APRIL 15

9:30–11:15 • Anabelian Geometry of Hyperbolic Curves, Progress on Section Conjecture

. . . . *Yuichiro* HOSHI *:: RIMS Kyoto University, Japan*

In this talk, we discuss some aspects of anabelian geometry. In the first part, we focus some anabelian results for hyperbolic curves over finite fields as an introduction to anabelian geometry. After a brief state of the background concerning fundamental notions in anabelian geometry, in order to provide elementary but nontrivial examples of basic arguments in the study of anabelian geometry, this part sketches briefly the idea of the proof of the anabelian result for hyperbolic curves over finite fields given by Tamagawa and Mochizuki. Here, recall that the case of finite base fields is a case to which anabelian reconstructions over number fields and function fields can be reduced. Moreover, the final portion reports on some recent progress in anabelian geometry over finite fields. We present how new approaches based on the l-adic cohomology theory and Kummer theory for algebraic curves, which are important ingredients in the arguments by Tamagawa and Mochizuki, give decisive progress in recent studies, e.g., the study of tripod-degrees.

In the second part, we focus on the study of Galois sections of hyperbolic curves. A Galois section of a hyperbolic curve over a field is defined to be a continuous section of the natural continuous surjective outer homomorphism from the etale fundamental group of the given curve to the absolute Galois group of the base field. Grothendieck's section conjecture states that, for a given hyperbolic curve over a number field, an arbitrary Galois section of the curve is geometric, i.e., the image of an arbitrary Galois section of the curve is contained in a decomposition subgroup associated to a closed point of the curve. In the second part, in particular, we discuss recent results concerning the geometricity of a Galois section of a hyperbolic curve over an arithmetic field.

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11:45 & 14:00 • Iwasawa theory and the geometry of the eigencurve

. . . . *Alexandre* MAKSOUD *:: Paderborn University, Germany*

L-functions encode deep arithmetic information about various mathematical objects, including elliptic curves and automorphic representations. Iwasawa theory provides powerful tools for studying the relationship between special values of L-functions and fundamental arithmetic invariants, such as class numbers of number fields and Mordell-Weil groups.

In the first part of the talk, I will discuss techniques from Iwasawa theory with applications to the equivariant Birch and Swinnerton-Dyer (BSD) conjecture. Through modularity theorems, one can reformulate BSD in terms of automorphic L-functions and leverage their properties to gain new insights.

The second part of the talk focuses on the p-adic variation of modular forms, encapsulated by the eigencurve, which plays a crucial role in results towards BSD. I will present joint work with Adel Betina and Alice Pozzi on the geometry of the eigencurve at p-irregular weight one forms, highlighting its significance in Iwasawa theory and its applications to the equivariant BSD conjecture.

References

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14:15 – 17:00 • Motivic cohomologies for qcqs schemes

. . . . *Shane* KELLY *:: The University of Tokyo, Japan*

While studying special values of L-functions, in the early 80's Beilinson and Lichtenbaum made several conjectures about the existence of a universal cohomology theory with integral coefficients referred to as *motivic cohomology*. In particular, it was conjectured that there should exist an Atiyah–Hirzebruch spectral sequence calculating algebraic K-theory from motivic cohomology. For smooth varieties over a field, this spectral sequence was subsequently constructed in multiple ways in work of Bloch, Grayson, Friedlander, Levine, Suslin, and Voevodsky.

Following the ideas used by Deligne in Hodge II to define mixed Hodge structures, Voevodsky defined motivic cohomology with compact support for not necessarily smooth varieties. He also defined a motivic cohomology for not necessarily smooth varieties—cdh motivic cohomology—whose corresponding Atiyah-Hirzebruch spectral spectral sequence calculates homotopy invariant algebraic K-theory.

In the first talk, aimed at non specialists, I will explain historical background about K-theory, motivic cohomology, and the cdh topology.

In the second talk, a usual research style talk, I will explain how to relax the cdh topology to a slightly coarser topology—the procdh topology—in order to produce a motivic cohomology for qcqs schemes. Procdh motivic cohomology sits in an Atiyah–Hirzebruch spectral sequence, and for Noetherian schemes this converges to algebraic K-theory. This is joint work with Shuji Saito.

References

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WEDNESDAY - APRIL 16

9:30–11:15 • Index formula of partially logarithmic characteristic cycles

. . . . *Yuri* YATAGAWA *:: Institute of Science Tokyo, Japan*

In this talk, we consider the computation of the Euler characteristic of a constructible sheaf on a smooth variety in terms of ramification theory. Ramification theory in arithmetic geometry is a study of relations between cohomological invariants of a constructible sheaf and invariants measuring the ramification of the sheaf. Following Deligne's observation for an analogy between the wild ramification of ℓ-adic sheaves in positive characteristic and the irregular singularity of partial differential equations on a complex manifold, we consider a construction of an algebraic cycle on the cotangent bundle for a constructible sheaf using ramification theory such that it computes the Euler characteristic as the intersection number with the zero section.

In the first part of the talk, we mainly recall the ramification theory. Especially, we recall invariants measuring the wild ramification of a smooth sheaf and explain relations with some ideas for the computation of the Euler characteristic given by Deligne, Kato, and Saito including the characteristic cycles.

In the second part, we construct an algebraic cycle on the logarithmic cotangent bundle with logarithmic poles along a subdivisor of the boundary for a smooth sheaf on a smooth variety following Kato's construction of the logarithmic characteristic cycle. After that, we give a formula for the Euler characteristic of the sheaf by using the algebraic cycle constructed with the invariants recalled in the first part.

References

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11:45 & 14:00 • Solid locally analytic representations

. . . . *Joaquin Rodrigues Jacinto :: Aix-Marseille University, France*

Abstract for the first part: Locally analytic representations were systematically studied by Schneider and Teitelbaum and play an important role in the p-adic Langlands correspondence, in the theory of p-adic L-functions and in the theory of families of automorphic forms. We will explain how condensed mathematics are useful for giving solid foundations of the theory of locally analytic representations. This is joint work with Juan Esteban Rodríguez Camargo

Abstract for the second part: I will report on the theory of solid locally analytic representations of a p-adic Lie group. I will explain how the category of solid locally analytic representation admits a description in terms of certain solid quasi-coherent sheaves over distribution algebras, generalising a well known result of Schneider and Teitelbaum. I will also discuss some applications to the p-adic Langlands correspondence.

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[RJRC] Joaquín Rodrigues Jacinto and Juan Esteban Rodríguez Camargo, Analytic D-modules and solid locally analytic representations *In Preparation*

14:15 – 17:00 • Anabelian reconstruction of Berkovich spaces through resolution of non-singularities and Grothendieck's p-adic anabelian conjecture

. . . . *Emmanuel* LEPAGE *:: IMJ-PRG, France*

Grothendieck conjectured that every Galois-compatible isomorphism of étale fundamental groups of hyperbolic curves over a number field should come from an isomorphism of the curves. After results of Nakamura and Tamagawa for affine curves, this conjecture was solved in the 90's by Mochizuki. More surprisingly, he proved it over a much wider class of fields, including p-adic fields. Neukirch-Uchida's theorem, saying that isomorphisms of absolute Galois groups of number fields come from isomorphisms of fields, makes the compatibility assumption over the absolute Galois group of the base field unnecessary in the number field case, but Neukirch-Uchida is false over p-adic fields. However, Mochizuki and Tsujimura proved in 2023 that the Galois-compatibility assumption is also unnecessary over p-adic fields.

Their proof relies on recovering the Berkovich space of the curve to get a group-theoretic characterization of Galois sections coming from rational points. In the first part of this talk, I will discuss the relation between the Berkovich space of the curve and Grothendieck's anabelian conjecture. One can first recover the combinatorics of the stable reduction of the curve. There is a natural map from the Berkovich space to the dual graph of this stable reduction, and by applying it to all finite étale covers, one gets a map from the universal pro-finite étale cover of the Berkovich space to an inverse limit of the dual graphs. Mochizuki and Tsujimura show that this map is bijective through resolution of non-singularities: for every small analytic disk in the curve, one can construct a finite étale cover of the curve whose preimage is not a union of disks.

In the second part of my talk, I will discuss the proof of resolution of non-singularities. By using a local criterion for $\mathbb{Z}_p(1)$-covers to exhibit such a behavior in terms of the vanishing of its Hodge-Tate log-differential, one reduces resolution of non-singularities to finding covers for which the Galois invariants of the cokernel of the Hodge-Tate log-differential is big enough. In the case, where the curve as ordinary reduction, the dimension of Galois invariants of the cokernel of the Hodge-Tate log-differential can be computed explicitly, and in the general case, Mochizuki and Tsujimura use constructions of Raynaud and Tamagawa to get covers with big ordinary quotients in their Jacobian variety.

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[Lep23a] Emmanuel Lepage, Resolution of Non-Singularities, *in MFO–RIMS Tandem Workshop: Arithmetic Homotopy and Galois Theory, B. Collas, P. Dèbes, Y. Hoshi, and A. Mézard, Eds., vol. 20, Oberwolfach Rep., EMS Press* **2023**, DOI: 10.4171/OWR/2023/42.

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THURSDAY - APRIL 17

9:30–11:15 • On Fargues-Scholze local Langlands correspondence for some supercuspidal representations of $Sp(6)$

. . . . *Yoichi* MIEDA *:: The University of Tokyo, Japan*

The local Langlands correspondence is a conjectural parametrization of irreducible representations of a p-adic reductive group by L-parameters, which is a variant of Galois representations. There are several approaches to this conjecture. For example, Arthur established the local Langlands correspondence for classical groups by using global automorphic technique. On the other hand, by using p-adic geometry, Fargues-Scholze attached a semisimple L-parameter to an irreducible representation of an arbitrary p-adic reductive groups. In this talk, I will compare these two correspondences in the case of Sp(6). In the first half, I will give a brief overview of current developments of the local Langlands correspondence. In the latter half, I will give a statement of our main theorem, and explain some ideas of the proof. This talk is based on a joint work with Masao Oi.

References

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11:45 & 14:00 • Eisenstein cocycles for imaginary quadratic fields

. . . . *Emmanuel* LECOUTURIER *:: Westlake University, China*

Ramanujan proved in 1916 the following congruence of power series in the variable q:

$$q\prod_{n\geq 1}(1-q^n)^{24} \equiv \frac{\zeta(-11)}{2} + \sum_{n\geq 1}\left(\sum_{d\mid n} d^{11}\right) q^n \text{ (modulo } p), \tag{1}$$

where $p = 691$ and $\zeta(s)$ is the Riemann zeta function. The two sides of (1) are the q-expansions of modular forms of weight $k = 12$ and level $\mathrm{SL}_2(\mathbf{Z})$. Recall that for $n \geq 2$ even, we have $\zeta(1-n) = -\frac{B_n}{n}$ where $B_n \in \mathbf{Q}$ is the nth Bernoulli number. In particular, from (1), we see that p divides the numerator of B_{12}. More precisely, the left-hand side Δ is a *cuspform* while the right-hand side E_{12} is an *Eisenstein series*, so (1) is called an *Eisenstein congruence.* The prime $p = 691$ is called an *Eisenstein prime* (in weight 12 and level 1).

It is natural to evaluate the L-series $L(\Delta, s)$ and $L(E_k, s) = \zeta(s)\zeta(s-k+1)$ associated to Δ and E_k at the *critical points* $s \in \{1, 2, ..., k-1\}$ (where $k = 12$) and ask whether, after suitable normalisation, these critical L-values are congruent modulo p. It turns out the normalisation will differ depending on the parity of s, so we have to consider separately the cases s even and odd.
For s even in $\{1, 2, ..., k-1\}$, $L(E_k, s)$ is (after suitable normalisation) a product of Bernoulli numbers, and the congruence of L-values hold. For s odd in $\{1, 2, ..., k-1\}$ and $s \neq 1, k-1$, we have $L(E_k, s) = 0$ and the congruence of L-values modulo p, while still true, is not as interesting.

Romyar Sharifi formulated beautiful conjectures, which in this example give a non-trivial congruence for $L(\Delta, s)$ (suitably normalised) modulo p for s odd in $\{1, 2, ..., k-1\}$ and $s \neq 1, k-1$. For reasons that we are not able to explain easily, in weight k and level $\Gamma_1(N)$, these congruences are described not directly in $\mathbf{Z}/p\mathbf{Z}$, but in the algebraic K-group $K_{2k-2}(\mathbf{Z}[\zeta_N])$ (where ζ_N is a primitive Nth root of unity). In our example, $k = 12$ and $N = 1$ so the relevant K-group is $K_{22}(\mathbf{Z})$, which is known to be cyclic of order 691.
In our talk, we shall restrict to the case of weight 2 and level $\Gamma_1(N)$. The Eisenstein prime $p = 691$ occuring in our previous example should be replaced by a certain *Eisenstein ideal* I of the corresponding Hecke algebra. The L-values of a cuspform are better described in terms of modular symbols, i.e. the singular homology group $H_1(X_1(N), \mathbf{Z})$, where N is

our level. Sharifi's conjecture relates $H_1(X_1(N), \mathbf{Z})$ to the algebraic K-group $K_2(\mathbf{Z}[\zeta_N])$ (*cf.* [Sha11]). More precisely, Sharifi defined a map $\varpi_N : H_1(X_1(N), \mathbf{Z}) \to K_2(\mathbf{Z}[\zeta_N])$ (after inverting 2) by sending the class of the geodesic path from $\frac{b}{d}$ to $\frac{a}{c}$ to the Steinberg symbol $\{1 - \zeta_N^c, 1 - \zeta_N^d\}$, where neither d nor c is divisible by N. He conjectured that ϖ_N is annihilated by our Eisenstein ideal I, which we refer to as the *Eisenstein property conjecture*, and that ϖ_N is an isomorphism modulo I (after inverting 2 and taking fixed parts for the action of the complex conjugation on both sides, which in our previous example corresponds to restricting to s odd).
Much results on the conjecture that ϖ_N factors through I have been obtained, in particular by [FK24] and [SV24]. The latter work of Sharifi and Venkatesh completely settles the Eisenstein property conjecture, away from Hecke operators in I at primes dividing N (for which Fukaya and Kato have partial results). The conjecture of Sharifi fits into the philosophy that the geometry of $\mathrm{GL}_2/\mathbf{Q}$ modulo Eisenstein relates to the arithmetic of $\mathrm{GL}_1/\mathbf{Q}$. The first part of our talk will be devoted to motivate and explain in details Sharifi's conjecture, as well as recalling some background.

It is natural to ask whether we can replace the base field $\mathbf{Q}$ by another number field K, in the sense that $X_1(N)/\mathbf{Q}$ should be replaced by a corresponding locally symmetric space for GL_2/K and $\mathbf{Z}[\zeta_N]$ by the ring of integers in a ray class field of K. A natural choice for K is an imaginary quadratic field, as in this case the Bianchi manifolds have a concrete description and, in special cases (e.g. when K is Euclidean), analogues of Manin symbols generate the H_1. In the second part of our talk, we will explain our joint work in progress with Romyar Sharifi, Sheng-Chi Shih and Jun Wang on the case where K is an *arbitrary* imaginary quadratic field. Our main result is the construction of an analogue of ϖ_N in this setting, and a proof of the Eisenstein property (at Hecke operators away from the level). We use the ideas and techniques developed by Sharifi and Venkatesh, by replacing the torus $\mathbf{G}_m^2/\mathbf{Q}$ by products of elliptic curves with CM by the ring of integers of K.

This is a joint work in progress with Romyar Sharifi, Sheng-Chi Shih and Jun Wang [LSSW].

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14:15 – 17:00 • Different approaches to Hyodo-Kato theory

. . . . *Veronika Ertl :: Caen University, France*

The classical de Rham theorem which compares Betti cohomology and de Rham cohomology for a smooth projective variety over the complex numbers has inpsired several comparison conjectures in the p-adic world [Fo94], which over the course of many years have become theorems due to the work of many people like Fontaine-Messing, Hyodo, Kato, Faltings, Tsuji, Nizioł, Beilinson, Bhatt, Scholze. In fact, a whole theory has developed out of this, called p-adic Hodge theory, with many recent developments in the realm of rigid analytic geometry.

One important building block of p-adic Hodge theory is Hyodo-Kato theory [HK94] Taking a historical perspective, I will explain the concept of Hyodo-Kato theory, and explain what role it plays in the above mentioned comparison theorems.

Then I will explain a rigid analytic approach to Hyodo-Kato theory (which is joint work with Kazukia Yamada) [EY24], which gives a very explicit construction, and explain how it related with other (more abstract) constructions.

References

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Friday - April 18

9:30–10:30 • Families preserving isomorphisms via techniques in anabelian geometry

.... *Koichiro Sawada :: RIMS Kyoto University, Japan*

For an isomorphism between closed subgroups of a profinite group G, if the image of any pro-cyclic subgroup I via this isomorphism is conjugate in G to I, then we shall say that this isomorphism is "families preserving".

Jarden and Ritter showed that, for a certain type of profinite group G —that includes the absolute Galois group of a p-adic local field and the étale fundamental group of hyperbolic curves—, every normal automorphism of G is inner. Their proof was divided into two steps: first showing that every normal automorphism is “families preserving” (in G), and then showing that every "families preserving" automorphism is inner.

In this talk, we discuss, from an anabelian geometrical point of view, whether a "families preserving" isomorphism between closed subgroups of a profinite group, such as the absolute Galois group of a certain field (Hilbertian field, Henselian discrete valuation field of positive residue characteristic) or its quotient, is induced from an inner automorphism. This is a joint work with Arata Minamide and Shota Tsujimura.

References

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[MST25] A. Minamide, K. Sawada, and S. Tsujimura, Families preserving isomorphisms via techniques in anabelian geometry (2025: in preparation, title to be confirmed).

10:45–12:30 • Reducibility of p-adic Banach principal series representations

. . . . *Noriyuki* Abe *:: The University of Tokyo, Japan*

I will discuss p-adic Banach representations of p-adic reductive groups. As usual in the theory of reductive groups, one can also define parabolic inductions in this case. However, the phenomenon is different from smooth representation theory. For example, we do not have Weyl group invariance.

In the talk, starting from basic theory of p-adic Banach representations, we will discuss parabolic inductions, especially focusing on the irreducibility of representations induced from finite dimensional representations.

This is based on a joint work with Florian Herzig.

References

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